# The origin of proton mass in Quantum Chromodynamics

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Quark masses in comparison with baryon masses

| | | | | |
|---|---|---|---|---|
| $p(uud)$ | $M_p \approx 940$ MeV | $m_u + m_d \approx 10$ MeV | comprises $\sim 1\%$ | in QCD |
| | $M_\Lambda \approx 1115\ MeV$ | $m_s = 105 - 150$ MeV | comprises $\sim 10\%$ | in QCD |

## Vacuum averages violating chirality

$$j_\mu^- = \bar{d}\gamma_\mu\gamma_5 u \qquad \langle 0 \mid j_{5\mu}^- \mid \pi^+\rangle = if_\pi^2 p_\mu$$

$$ip_\mu\langle 0 \mid j_{5\mu}^- \mid \pi^+\rangle = f_\pi m_\pi^2$$

$$iq_\mu(m_u + m_d)\int d^4x e^{iqx}\langle 0 \mid T\left\{j_{\mu 5}^-(x),\ \bar{u}(0)\gamma_5 d(0)\right\} \mid 0\rangle \qquad \text{quark masses}$$

$$j_{\mu 5}^- = \bar{d}\gamma_\mu\gamma_5 u \qquad \text{are neglected}$$

$$-(m_u+m_d)\int d^4x e^{iqx}\langle 0 \mid \delta(x_0)\langle 0 \mid \left[j_{05}^{-}(0),\ \bar{u}(0)\gamma_5 d(0)\right] \mid 0\rangle =$$

$$= (m_u + m_d)\langle 0 \mid \bar{u}u + \bar{d}d \mid 0\rangle$$

$$q_\mu\langle 0 \mid j_\mu^- \mid \pi^+\rangle\left(-\frac{1}{q^2}\right)\langle \pi^+ \mid (m_u + m_d)\bar{u}\gamma_5 d \mid 0\rangle = -f_\pi^2 m_\pi^2$$

$$\langle 0 \mid \bar{q}q \mid 0\rangle = -\frac{1}{2}\frac{m_\pi^2 f_\pi^2}{m_u + m_d} = -(254\ \text{MeV})^3 \ \ \text{(at 1 GeV)}$$

## Lagrangian QCD

$L$(quarks) $\quad L_Q = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) \quad \partial = \gamma_\mu \partial_\mu = \gamma_0 \frac{\partial}{\partial t} + \vec{\gamma} \frac{\partial}{\partial \vec{r}}$

$$\not{\partial}_\mu \equiv \partial_\mu + igA_\mu^a \frac{\lambda^a}{2}$$

$L$(gluons) $\quad L = \frac{1}{4} G_{\mu\nu} G_{\mu\nu}$

$$G_{\mu\nu}^a = -\frac{i}{g}\left[\not{\partial}_\mu, \partial_\nu\right] = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + igf^{aik}\left[A_\mu^i(x),\ A_\nu^k\right]$$

## Euclidean space

$$x_0 \to -ix_y \quad \partial_\mu = \left(\frac{\partial}{\partial x_0},\ \frac{\partial}{\partial x_i}\right),$$

$$p_\mu = -i\frac{\partial}{\partial x_\mu}, \quad A_i^E = -A_i^{Mink}, \quad A_4^E = -iA_0^{Mink}$$

$$\bar{\psi} \to \psi^+(!) \qquad \left\{\psi(0), \psi^+(0)\right\} = 0$$

# Instantons and topological quantum numbers

Euclidian space

$$S = \frac{1}{4}\int d^4x G^n_{\mu n} G^n_{\mu\nu} = \frac{1}{4}\int d^4x \left[ G^n_{\mu\nu}\tilde{G}^n_{\mu\nu} + \frac{1}{2}\left( G^n_{\mu\nu} - \tilde{G}^n_{\mu\nu} \right)^2 \right]$$

$$\tilde{G}^n_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma} G^n_{\lambda\sigma} \qquad\qquad \varepsilon = 1234 = 1$$

$$S_{min} = \frac{1}{4}\int d^4x G^n_{\mu\nu}\tilde{G}^n_{\mu\nu}$$

$$G^n_{\mu\nu}\tilde{G}^n_{\mu\nu} = \partial_\mu K_\mu$$

$$K_\mu = 2\varepsilon_{\mu\nu\gamma\delta}\left( A^n_\nu \partial_\gamma A^n_\delta - \frac{1}{3} g f^{nmp} A^n_\nu A^m_\gamma A^0_\delta \right)$$

$$S_{min} = \frac{1}{4} \int dV K_4$$

$$S_{min} = -\frac{1}{12} g \varepsilon_{ikl} \int dV f^{nmp} A_i^n A_k^m A_l^p$$

$S(2)$ – subgroup $\qquad f^{nmp} \to \varepsilon^{nmp}$

$$A_i = -\frac{1}{2} g \tau^n A_i^n$$

$$Tr(\tau^n \tau^m) = \delta_{nm}$$

$$S_{min} = -\frac{1}{3} \frac{1}{g^2} \int dV \varepsilon_{ikl} Tr(A_i A_k A_l)$$

$$A_i' = U^{-1} A_i U + i U^{-1} \partial_i U$$

$$S_{min} = -\frac{1}{3} g^2 \int dV \varepsilon_{ikl} Tr\left[\partial_i U^{-1} \partial_k U^{-1} \partial_l U\right]$$

$A_i(r)$ valishes faster, than $1/r$, at $r \to \infty$: $U$ – is a constant matrix

$$U = \exp(\theta^a \tau^a / 2i)$$

$$S_{min} = -\frac{1}{2} \varepsilon_{ikl} \varepsilon^{abc} \frac{1}{g^2} \int dV \partial_i \left[\tilde{\theta}^a \partial_k \tilde{\theta}^b \theta_l \tilde{\theta}^c \left(\sin\theta - \theta\right)\right]$$

$$\theta = \sqrt{\theta^a \theta^a} \qquad \tilde{\theta}^a = \theta^a / \theta$$

$$U = \pm 1$$

$$\theta = \pm 2\pi n, \qquad r \to \infty$$

$$\boxed{S_{min} = \frac{8\pi^2 n}{g^2} = \frac{2\pi}{\alpha_s} n}$$

$$G_\mu = \tilde{G}_\mu; \text{ group } SU(2)$$

$$A_\mu^n(x) = -\frac{2}{g} \eta_{a\mu\nu} \frac{x_\nu}{x^2 + \rho^2}$$

$$G_{\mu\nu}^n = \frac{4}{g} \eta_{a\mu\nu} \frac{\rho^2}{(x^2 + \rho^2)^2}$$

$$\eta_{a\mu\nu} = \begin{cases} \varepsilon_{a\mu\nu} & \mu, \nu = 1, 2, 3 \\ -\delta_{a\nu} & \mu = 4 \\ \delta_{a\mu} & \nu = 4 \\ \eta_{a44} = 0 & \end{cases}$$

## Fermions in the instanton field

$$-i\gamma_\mu \bigtriangledown_\mu \psi_n = \lambda_n \psi_n \quad \bigtriangledown_\mu = \frac{\partial}{\partial x_\mu} + ig\frac{\lambda^k}{2}A_\mu^k \quad k = 1, ...8$$

$$-det(i\gamma_\mu \bigtriangledown_\mu = \Pi_n(\lambda_n - im)$$

$$\hat{\bigtriangledown} = \gamma_\mu \bigtriangledown_\mu$$

$$L = -d^4x\left[\psi_L^+ \hat{\bigtriangledown}\psi_R + \psi_R^+ \hat{\bigtriangledown}\psi_L\right]$$

$$\lambda_n \neq 0 \quad L = -\int d^4x\left[\psi_L^+ \bigtriangledown \psi_R + \psi_R^+ \bigtriangledown \psi_L\right] - \text{Chirality inv}$$

$$\lambda_n = 0 \quad L = -\int d^4x\left[\psi_L^+ + \psi_R^+\right]\hat{\bigtriangledown}_\mu -$$

no conclusion can be done about chirality invariance

Källen-Lehman representation

$$\hat{\bigtriangledown} S(x) = \delta(x) \ \ TrS(x) = \frac{1}{\pi}\int dx\rho(\lambda)\Delta(x^2,\lambda)$$

$S(x)$ – fermions propagator

$$x^2 \to 0 \quad \triangle(x^2) \to \delta(\lambda) \quad \rho = \langle 0 \mid \psi^+(0)\psi(0) \mid 0\rangle = \ TrS(x)$$

$\lambda_n \neq 0 \to \gamma_5\psi$ – is also the solution of Dirac equation, corresponding $\lambda n' = -\lambda_n$

Banks, Casher

$$-i\gamma_\mu\nabla_\mu u = 0 \qquad \gamma_4\left[\sigma_\mu^+\bigtriangledown_\mu(1{+}\gamma_5)\chi_L{+}\sigma_\mu^-\bigtriangledown_\mu(1{-}\gamma_5)\chi_R\right] = 0$$

$$u = \frac{1}{2}\left(1{+}\gamma_5\right)\chi_L{+}\frac{1}{2}(1{-}\gamma_5)\chi_R \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_\mu^\pm = (\vec{\sigma} \mp i)$$

$$\sigma_\nu \times \bigtriangledown_\nu \times \left[ \sigma_\mu^+ \bigtriangledown_\mu \chi_L = 0 \right] \qquad - \bigtriangledown_\mu^2 \chi_L = 0$$

$$\sigma_\mu^+ \bigtriangledown_\mu \left( \sigma_\mu^- \bigtriangledown_\mu \chi_R \right) = 0 \qquad - \bigtriangledown_\mu^2 \chi_L = -4 \boldsymbol{\sigma} \boldsymbol{\tau} \frac{\rho^2}{(x^2 + \rho^2)^2} \chi_R$$

$\tau_\mu^\pm -$ isospin matrices

$$\left[\nabla_\mu, \nabla_\nu\right] = i\frac{g}{2}\tau^a G^a_{\nu\mu}$$

$$u_{0R} = \frac{1}{2}\left(1 - \gamma_5\right)\frac{1}{\pi}\frac{\rho}{(x^2 + \rho^2)^{3/2}}\chi_0,$$

$$\chi_0\left[\chi_s\left(\frac{1}{2}\right)\chi_0\left(-\frac{1}{2}\right) - \chi_s\left(-\frac{1}{2}\right)\chi_c\left(\frac{1}{2}\right)\right]$$

## Proton mass

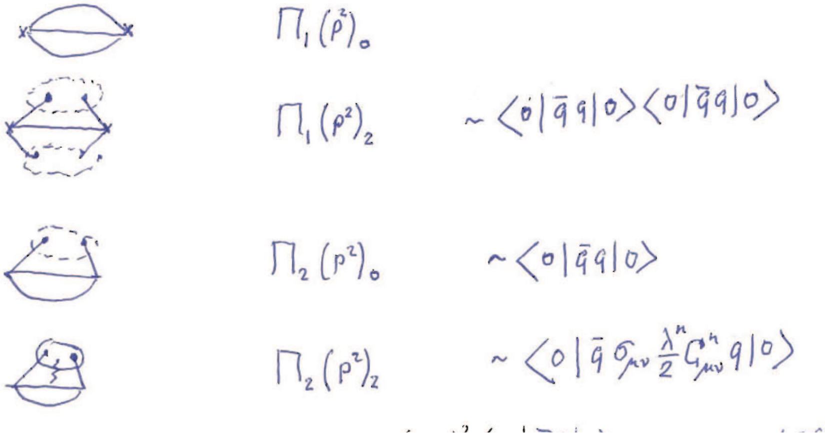
$$\Pi^{AB}(x) = iT\left\{j^A(x), j^B(0)\right\}_{x\to 0} = \Sigma_n C_n^{AB}(x) O_n(0)$$

$$m_p^3 = -C\langle 0 \mid \bar{q}q \mid 0\rangle$$

$$\Pi(p) = i\int d^4x e^{ipx}\langle 0 \mid T\{\eta(x),\ \bar{\eta}(0)\} \mid 0\rangle$$

$$\eta(x) = 2\left[(u^a C d^b)\gamma_5 u^c + (u^a C\gamma_5 d^b)u^c\right]\varepsilon^{abc}$$

$$\Pi(p) = \not{p}\Pi_1(p^2) + \Pi_2(p^2)$$




$$(m_p^3)_0 = -2(2\pi)^2 \langle 0|\bar{q}q|0\rangle \qquad m_0 = 1.09 \text{ GeV}$$

$$(m_p) = 0.98 \pm 0.10 \text{ GeV}$$

**The existance of Higgs boson has nothing to do with proton mass.**